©2016 Published in 4th International Symposium on Innovative Technologies in Engineering and Science 3-5 November 2016 (ISITES2016 Alanya/Antalya - Turkey)

Impact of Replenishment Schedules When a Vendor Manages Inventory at Multiple Retailers with Storage Limits

*Mehmet Gümüş
*Faculty of Engineering, Department of Industrial Engineering, Alanya Alaaddin Keykubat University, Antalya, Turkey

Abstract

We study a supply chain in which a single vendor manages multiple retailers' inventory levels under a vendor managed inventory (VMI) contract. The contract specifies stock limits at the retailers' facilities. We formulate the problem as a nonlinear mixed integer model that minimizes the total cost in the entire supply chain. As the model is difficult to solve, we provide an approximation method to find a solution, and construct sets of extensive numerical examples for sensitivity analysis. We generate a total of 5400 example problems and use them to assess the impacts of various model parameters on supply chain savings under VMI. Results suggest that the vendor and retailers should negotiate for VMI contracts that do not dictate tight upper stock limits at the retailers, although the retailers might want such restrictions to safeguard themselves.

Key words: Vendor managed inventory, supply chain, inventory replenishment

1. Introduction

In today's competitive markets, supply chain companies need to coordinate their decisions in order to improve customer service and maximize profits [1]. Vendor managed inventory (VMI) is a partnership that achieves coordinated decision making between a vendor and its customer [2]. Under VMI, the retailers share their sales and stock information with their vendor who in turn manages the replenishments and inventories at the retailers. Sometimes, VMI is coupled with consignment inventory via which the vendor assumes ownership of the goods until they are sold by the retailers [3]. In this paper, we consider a supply chain that includes a single vendor who is a distributor, and multiple retailers. The vendor manages the retailers' stock levels under a VMI agreement that does not require consignment stock. In the VMI agreement, the retailers set upper stock limits at their premises. The vendor is charged a penalty cost for exceeding those stock limits. Demand is deterministic and realized that the retailers. In any given cycle of the vendor, retailers replenishment frequencies may be unequal, hence we don't force equal replenishment cycles for the retailers.

Our analyses relate both to the coordination aspect of VMI and the operational benefits it enables. VMI as a means of channel coordination has been studied by researchers such as Bernstein and Federgruen [4]. The operational benefits, such as delivery flexibility, that VMI may create for the vendor can result from combining routes from multiple origins, consolidating shipments to two or more customers, and allowing the supplier to construct better delivery

*Corresponding author: Address: Faculty of Engineering, Department of Industrial Engineering, Alanya Alaaddin Keykubat University, 07425 Antalya TURKEY. E-mail address: mehmet.gumus@alanya.edu.tr, Phone: +902425182266

schedules for multiple retailers. For example, Cetinkaya and Lee [5] analyze how a vendor under VMI can synchronize inventory and transportation decisions by holding orders until a suitable time to dispatch a consolidated load. Chaouch [6] assumes that the vendor can change shipment frequencies to balance shortage cost, and obtains the vendor's shipment rate under VMI. An overview of benefits of VMI and CI for multiple retailers when compared to a no-VMI is provided by Hariga et al. [7].

In our study, we assess the benefits of VMI for the supply chain when retailers set stock limits and penalty costs for exceeding those limits. We develop a mixed integer nonlinear program and solve it using a heuristic. We provide various analysis through numerical examples to discuss when a VMI agreements generate more benefits for the supply chain.

2. Problem Formulation

Let m be the number of retailers, and j be the index for retailers, j =1, 2, …, m . Parameters are

H_v holding cost per unit per unit of time at the vendor.

A_v ordering cost for the vendor.

d_j demand at retailer j and $D_v = \sum_{j=1}^{n} d_j$, vendor's total demand.

a_j cost per order for retailer j

h_j' holding cost per unit per unit of time at retailer j

z_j Penalty cost incurred by the vendor when the inventory level at the jth retailer exceeds the maximum allowed inventory.

I_j^{max} = maximum inventory allowed by retailer j.

h_j $= h_j' - H_v$, echelon holding cost for retailer j.

Decision variables are

T_j cycle length for retailer j.

T_v cycle length for the vendor.

n_j number of shipments made by the vendor to retailer j during T_v .

q_j $= T_j d_j$, replenishment quantity for the jth retailer.

Our models will extend the analysis provided by Darwish and Odah [8] where they model a VMI for a single vendor and multiple retailers with stock limits. However, we do assume, as they do, that replenishment intervals are the same for all supply chain members. In their paper, they provide a total cost function that includes a total penalty cost for excessive stock at the retailers. It is not difficult to show that the total over-stocking penalty cost in the supply chain is $\sum_{j=1}^{n} z_j \frac{\tilde{I}_j^{\,2}}{2T_j d_j}$ where $\tilde{I}_j = Max(0, q_j - I_j^{max})$, the over-stock quantity at the jth retailer. We can now provide the optimization model for the supply chain under VMI.

Problem Z: Minimize

$$TC = \frac{A_v}{T_v} + \sum_{j=1}^{m} \frac{a_j}{T_j} + \frac{H_v D_v T_v}{2} + \frac{1}{2}\sum_{j=1}^{m} h_j d_j T_j + \sum_{j=1}^{m} \frac{z_j \tilde{I}_j^2}{2T_j d_j}. \quad (1)$$

s.t.

$$T = n_j T_j \text{ for } j = 1, 2, \ldots, m \quad (2)$$

$$T_j D_j - \tilde{I}_j \le I_j^{max} \text{ for } j = 1, 2, \ldots, m \quad (3)$$

m_j integer; $T_v,\ T_j$ and $z_j \geq 0$ for $j = 1, 2, \ldots, m$

In the total cost function, the first two terms are ordering costs, the next two terms are holding costs based on echelon stocks, and the last term is the overstock penalty cost. Constraint (2) ensure that the the vendor cycle is an integer multiple of a retailer's cycle. Finally, since $\tilde{I}_j \geq 0$, constraint (3) ensure that $\tilde{I}_j = Max\left(0, q_j - I_j^{max}\right)$.

3. Solving the Problem

The model for problem Z is a mixed integer nonlinear model, which may be difficult to solve for large problem sizes. To obtain a near optimal solution, we first relax problem P and solve the relaxed version to generate a lower bound to problem P. In the relaxed version, constrained (2) is replaced by $T_v \geq T_j$ for $j = 1, 2, \ldots, m.$ Everything else remains the same as in Problem Z.

First of all, we solve the relaxed problem to compute a total cost as the lower bound. Also, we compute the continuous shipment frequencies and the vendor's cycle from the relaxed solution, which form the starting values. After solving the relaxed problem and finding the lower bound, we use that solution to generate a feasible solution which forms the upper bound. The steps we follow in generating a solution are explained below.

Step 1: Solve the relaxed version of problem Z. As the relaxed problem does not include any integer variables, any commercial software can easily find a solution.

Step 2: Solution of the relaxed problem provides a T_v and T_j for $j = 1, 2, \ldots, m$. As there is no integer requirement in the relaxed problem, T_v / T_j = n_j is continuous. Then,

for j = 1, 2, …, k-1, set $\lfloor n_j^{Relaxed} \rfloor \leq n_j \leq \lceil n_j^{Relaxed} \rceil$. Note that $\lfloor x \rfloor$ and $\lceil x \rceil$ are the largest integer smaller than *x*, and smallest integer larger that *x*, respectively.

Step 3: Solve problem Z with $\lfloor n_j^{Relaxed} \rfloor \leq n_j \leq \lceil n_j^{Relaxed} \rceil$ which limits the values of $n_j$to only two integers.

4. Numerical Examples and Analysis

In this section, we show the results of our sensitivity analysis conducted to assess the impact of various model parameters on supply chain performance under VMI. In solving problems, we used Cplex 12.2 in a computer with 2.70GHz processor and 4GB RAM. Two types of measures were used in our analysis. A percentage gap between the solutions to relaxed problem and original problem (Z), and a percentage savings achieved by *VMI* in (Z) compared to the total supply chain costs without *VMI*. Total supply chain costs without VMI were approximated using what is proposed by Darwish and Odah [8].

We solved a total of 5400 problems which included 360 instances for each $m \in \{10, 20, .., 140, 150\}$. The remaining problem parameters were generated randomly from uniform distributions over the ranges given in Table 1. We considered three classes of demand rates: low, medium, and high. We also experimented with three sets of penalty cost: low, medium, and high. Finally, we studied the effect of tight and moderate upper stock levels set by the retailers.

Table 1. Experimental settings for the problem parameters.

Parameter	Range [lower limit, upper limit]
Vendor's ordering cost, A_v	[3500, 4000]
Vendor's holding cost, H_v	[0.5, 1.5]
Retailer's ordering cost, a_j: j= 1,2,…, m	$[10, A_v/10]$
Retailer's holding cost, h_j': j= 1,2,…, m	$[2H_v, 10H_v]$
Demand rate, d_j: j= 1,2,…, m	Low:[100,500], Medium: [500,10000] High: [10000,100000]
Penalty Cost, z_j: j= 1,2,…, m	Low: [1,5], Medium: [5,10], High: [10,20]
Stock Limit, I_j^{max}: j= 1,2,…, m	Tight: $I_j^{max} = EOQ_j$, Moderate: $[EOQ_j, 1.5EOQ_j]$

All possible combinations of the experimental settings given in Table 1 were considered. Hence, we generated a total of 270 different test problems, each of which was replicated 20 times using different seeds. For each of the 5400 problems solved, we calculated the percentage gap and found the percentage savings. For each measure type and for each test problem, we then calculated an average and recorded the maximum and the minimum observed per 20 replications.

Of all the problems solved, the average gap is 1.63%. The minimum, 0.073%, is observed when m=10, d_j is high, I_j^{max} is moderate, and z_j is medium. We record the maximum 4.3% when m=60, d_j and z_j are high, and I_j^{max} is tight. On the other hand, average cost savings over all the problems solved is 4.59%. The maximum savings of 9.38% is recorded when m=20, d_j and z_j are medium, and I_j^{max} is moderate. The lowest savings, 0.923%, is observed when m=120, d_j and z_j are high, and I_j^{max} is tight.

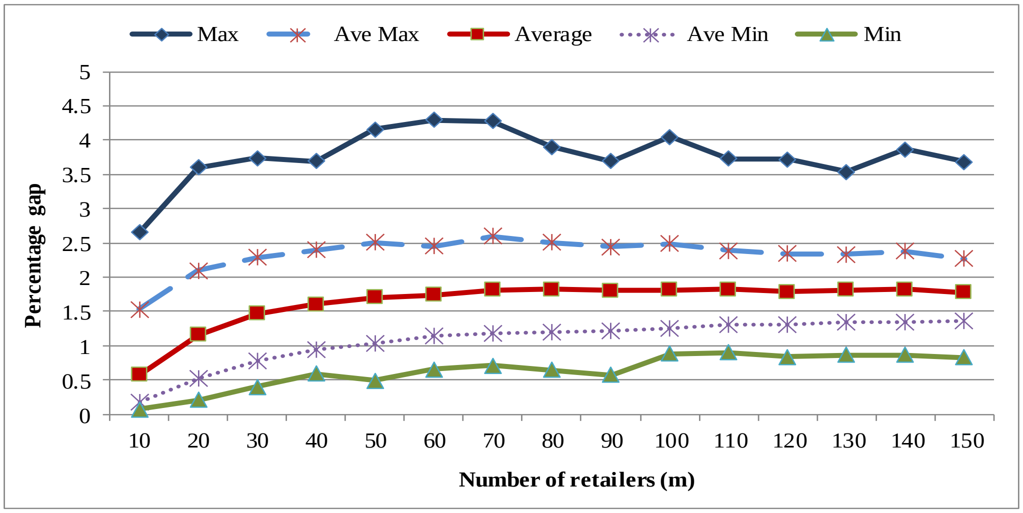

Figure 1. Percentage gap per number of retailers.

Figure 1 shows the summary results per *m*, the number of retailers in the problem, over all the parameter settings considered. For each *m*, the average gap found in 180 problems (18 settings replicated 20 times), the maximum and the minimum gaps observed are depicted with lines Average, Max and Min, respectively. The lines Ave Max and Ave Min show the averages of 18 maximum and minimum gaps recorded for each setting. Results suggest that as we increase *m*, the average gap increases first, and then becomes stable at around 1.8%. The lines Ave Max and Ave Min support this finding since they converge towards 1.8% as *m* increases.

The results above are reasonable. For smaller problem sizes, the relaxed problem generates a lower bound solution close to the one obtained by our heuristic which constrains the vendor's delivery cycle to be multiple of the retailers' reorder intervals. The restriction ($T_v = n_j T_j$) on the delivery schedules has more impact on the cost performance of our heuristic for $m \geq 70$ since the delivery coordination becomes more difficult for large number of retailers. This suggests that all problems with $m \geq 70$ may be categorized as large-size problems where the percentage gap is highest, but not more than 1.8% on the average.

Table 2 is used to summarize the percentage gaps found in all experimental settings over all problem sizes when the retailers' allowed stock limit, I_j^{max}, is set to be tight and moderate.

Table 2: Overall percentage gaps per upper stock limit and demand settings.

Demand Category	Average, [Min, Max]:	Stock Limit Category	Average, [Min, Max]:
Low d_j	1.77, [0.08, 4.28]	Tight I_j^{max}	2.12, [0.20, 4.28]
		Moderate I_j^{max}	1.42, [0.08, 3.50]
Medium d_j	1.41, [0.11, 3.13]	Tight I_j^{max}	1.62, [0.14, 3.13]
		Moderate I_j^{max}	1.20, [0.11, 2.51]
High d_j	1.72, [0.07, 4.30]	Tight I_j^{max}	2.03, [0.17, 4.30]
		Moderate I_j^{max}	1.41, [0.07, 3.11]

The results in Table 2 suggest that the average percentage gap under medium demand quantities at the retailers is lower than the average percentage gap under low or high demand. It also supports our discussions on Figure 3 that low penalty costs generate a lower percentage gap on the average compared to medium or high penalty costs. A tight upper limit on the retailers' stock level, on the other hand, increases the percentage gap for a given demand setting compared to a moderate upper stock limit under the same setting.

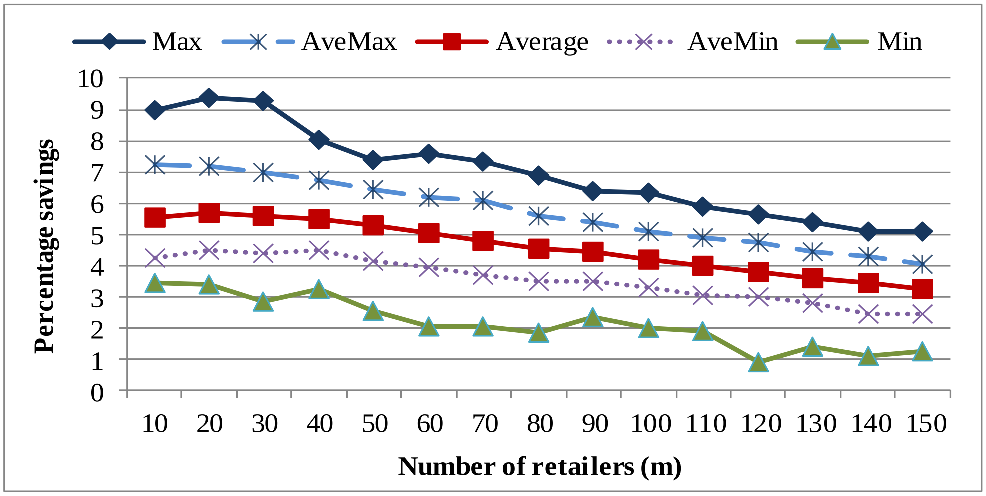

Figure 2. Percentage savings per number of retailers.

Let us now look at the percentage savings generated with *VMI*. Figure 2 shows the average, the maximum and average of maximums, and the minimum and average of minimums over all experimental settings when number of retailers in the supply chain, *m*, varies between 10 and 150. As that number increases, percentage savings tend to decrease on all lines. Average percentage savings is the highest when *m*=20, and it is the lowest when *m*=150. There are lower percentage savings for larger *m* values because if *m* is large, it is more difficult to coordinate the supply chain, and the total storage penalty cost over all retailers' sites is high as there is more excessive stock.

Table 3. Percentage savings per upper stock limit and demand settings.

Demand Category	Average, [Min, Max]:	Stock Limit Category	Average, [Min, Max]:
Low dj	4.79, [1.74, 9.27]	Tight I_j^{max}	4.34, [1.74, 8.03]
		Moderate I_j^{max}	5.24, [3.07, 9.27]
Medium dj	4.47, [1.12, 9.38]	Tight I_j^{max}	4.17, [1.12, 7.52]
		Moderate I_j^{max}	4.78, [1.97, 9.38]
High dj	4.50, [0.92, 8.06]	Tight I_j^{max}	4.06, [0.92, 7.18]
		Moderate I_j^{max}	4.96, [2.32, 8.06]

Table 3 shows the impact of the parameter settings over all the problems sizes considered. We see in Table 3 that percentage savings tend to be the highest under low d_j, followed by high d_j category which gives slightly better average savings compared to the medium d_j category.

Since a *VMI* contract permits a vendor to order on behalf of its retailers, those retailers may choose to set tight restrictions on the maximum allowed shipment sizes from the vendor. They may also prefer to lower their holding costs by charging the vendor high penalty costs whenever the allowed stock limits are exceeded. However, our analysis show that such actions of the retailers are less beneficial for the supply chain as a whole: Supply chain cost savings tend to be greater with *VMI* contracts which allow larger stock limits at the retailers and include low penalty costs for exceeding those limits. Moreover, the results on percentage savings suggest that there are potential cost reductions in a supply chain under *VMI* regardless of the number of retailers involved. Those savings can then be shared among all the supply chain members to guarantee benefits for each member compared to a *non-VMI* supply chain.

5. Conclusions

We analyzed a supply chain in which a vendor controls the stock levels of multiple retailers using a VMI contract. With maximum stock levels at the retailers and penalty cost for the vendor for exceeding those, we modeled the supply chain problem as a mixed integer nonlinear program. Upper and lower bounds for the resulting optimization problem were obtained using simple heuristics. We solved a total of 5400 problems to conduct sensitivity analysis. The results showed the impacts of various parameters on the percentage gap between the lower bound and our heuristic solutions, and also on the percentage savings in total supply chain cost under *VMI* compared to non-*VMI*. *VMI* contracts with low levels of allowed upper-stock limits at the retailers, and medium to high penalty costs for exceeding those limits result in larger deviations between the solutions to the relaxed problem and the original problem Z. Moreover, the percentage gap for smaller problem sizes may be as low as 0.073%. That gap increases as we increase *n* until we reach the category of large size problems. For all those large size problems, the gap stays around 1.8 % on the average. Overall gap attained was 1.63% on the average, and varied in the range [0.073%, 4.299%].

Cost savings over all the problems solved was 4.59% on the average, and ranged between 0.92% and 9.38% over all the experimental settings. As the number of retailers in the supply chain increased, overall percentage savings decreased. The analysis on percentage savings showed also that *VMI* contracts that specify tighter restrictions on upper stock limits result in less cost reductions on the average for the supply chain. If those restrictions are coupled with medium to high penalty costs, savings are even lower. These results suggest that the involved parties should negotiate a *VMI* contract that does not dictate high penalty costs and tight upper stock limits, although the retailers might want such restrictions to safeguard themselves.

References

[1] Brown JR, Dant RP, Ingene CA, Kaufmann PJ. Supply chain management and the evolution of the "Big Middle". Journal of Retailing 2005;81(2):97–105.

[2] Bookbinder JH, Gumus M, Jewkes EM. Calculating the benefits of vendor managed Inventory in a manufacturing-retailer system. International Journal of Production Research 2010;48:5549–5571.

[3] Gumus M., Jewkes EM, Bookbinder JH. Impact of consignment inventory and vendor managed inventory for a two-party supply chain. International Journal of Production Economics 2008;113: 502–517.

[4] Bernstein F, Federgruen A. Pricing and replenishment strategies in a distribution system with competing retailers. Operations Research 2003;51:409–426.

[5] Cetinkaya S, Lee CY. Stock replenishment and shipment scheduling for vendor-managed inventory systems. Management Science 2000;46(2):217–232.

[6] Chaouch BA. Stock levels and delivery rates in vendor-managed inventory programs. Production and Operations Management 2001; 10/1: 31–44.

[7] Hariga M, Gumus M, Ben-Daya M, Hassini E. Scheduling and lot-sizing models for the single vendor multi-buyer problem under consignment stock partnership. Journal of the Operational Research Society 2013;64/7: 995–1009

[8] Darwish MA, Odah OM. Vendor managed inventory model for single- vendor multi-retailer supply chains. European Journal of Operational Research 2010;204(3):473–484.